

# Agreement among mobile robots in the three-dimensional Euclidean space: The plane formation problem and the pattern formation problem

Yukiko Yamauchi \*

Kyushu University, Japan.

**Abstract.** We consider a swarm of autonomous mobile robots each of which is anonymous and oblivious (memory-less), and synchronously executes the same algorithm. The *plane formation problem* requires the robots to land on a common plane without forming any multiplicity from a given initial configuration and the *pattern formation problem* requires the robots to form a given target pattern from an initial configuration. We first investigate the pattern formation problem for oblivious fully-synchronous (FSYNC) robots moving in the three dimensional Euclidean space (3D-space), and characterize the problem by showing a necessary and sufficient condition for the robots to form a target pattern  $F$  from an initial configuration  $P$ . The pattern formation problem in the two dimensional Euclidean space (2D-space) has been characterized by Yamashita and Suzuki (TCS 2010) and Fujinaga et al. (SICOMP 2015). They showed a necessary and sufficient condition based on the notion of *symmetricity* of an initial configuration that shows the symmetry that the robots can never break. The symmetricity of  $\rho(P)$  of positions of robots  $P$  is intuitively the order of the cyclic group of the initial configuration. It has been shown that the oblivious FSYNC robots can form a target pattern  $F$  from an initial configuration  $P$  if and only if  $\rho(P)$  divides  $\rho(F)$ . We extend the notion of symmetricity to 3D-space by using the *rotation groups* that is defined by a set of rotation axes and their arrangement. We define the symmetricity  $\varrho(P)$  of positions of robots in 3D-space as the set of rotation groups formed by rotation axes that the robots can never eliminate. We show the following necessary and sufficient condition for the pattern formation problem which is a natural extension of existing results of the pattern formation problem in 2D-space: The oblivious FSYNC robots in 3D-space can form a target pattern  $F$  from an initial configuration  $P$  if and only if  $\varrho(P) \subseteq \varrho(F)$ . We will show the impossibility by showing the worst case arrangement of local coordinate systems of robots, that does not allow the robots to break their symmetricity. For the possibility cases, we present a pattern formation algorithm for oblivious FSYNC robots. As a corollary of the main result, we can show a necessary and sufficient condition for the robots to form a plane: The oblivious FSYNC robots in 3D-space can form a plane if and only if  $\varrho(P)$  is a cyclic group or a dihedral group. We then show a plane formation algorithm for oblivious FSYNC robots.

**Keywords.** FSYNC model, mobile robots in three dimensional Euclidean space, plane formation problem, pattern formation problem, rotation group, symmetry breaking.

---

\* Corresponding author. Address: 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan. Fax: +81-92-802-3637. Email: yamauchi@inf.kyushu-u.ac.jp