Asynchronous enzymatic numerical P systems for the compare-and-exchange and sorting

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Abstract

We consider asynchronous EN P systems, which are computational models inspired by the structures and behaviors of living cells, for compare-and-exchange and sorting. We first propose an asynchronous EN P system for the compare-and-exchange, and show that the asynchronous EN P system works in $O(1)$ sequential and parallel steps. We next propose an asynchronous EN P system for sorting of n numbers, and show that the asynchronous EN P system works in $O(n)$ parallel steps and $O(n^2)$ sequential steps.

1. Introduction

A number of next-generation computing paradigms have been considered due to limitation of silicon-based computation. In the next-generation computing paradigms, natural computing, which works using natural materials for computation, has considerable attention. As one of the natural computing, a numerical P system, which is inspired from structures of living cells and economics, has been introduced in [1]. In addition, an enzymatic numerical P system [2] (EN P system, for short) is also a model such that a variable called enzyme is used to promote evolution programs.

A number of EN P systems have been proposed for some operations. For example, an EN P system for the compare-and-exchange has been proposed in [3]. However, synchronous application of programs is assumed in the EN P system with maximum parallelism. The maximal parallelism means that all applicable programs are applied synchronously.

On the other hand, there is obvious asynchronous parallelism in the cell biochemistry. The asynchronous parallelism means that all programs are independently applied with different speed. Since all objects in a living cell basically works in asynchronous manner, the asynchronous parallelism must be considered to make the EN P system more realistic model.

In the present paper, we first propose asynchronous EN P system for the compare-and-exchange, and show that the asynchronous EN P system works in $O(1)$ parallel steps and $O(1)$ sequential steps.

We next propose an asynchronous EN P system for sorting n numbers using the asynchronous EN P system for compare-and-exchange as sub-systems. The asynchronous EN P system works in $O(n)$ parallel steps and $O(n^2)$ sequential steps.

2. Asynchronous EN P system

The EN P systems and the sets used in the system are defined as follows.

$$\Pi_{ENP} = (m, H, \mu, (V_1, P_1, V_1(0)), (V_2, P_2, V_2(0)), \cdots, (V_m, P_m, V_m(0)), V_O))$$

- m : m is the number of membranes.
- H: H is a set of labels for membranes. (We assume that a membrane labeled 1, which is called the skin membrane, is the outermost membrane, i.e., the skin membrane contains all of the other membranes.)
- μ: μ is membrane structure that consists of m membranes.
- V_i: V_i is a set of numerical variables in the membrane labeled i.
- P_i: P_i is a set of evolution programs in the membrane labeled i .
- $V_i(0)$: $V_i(0)$ is a set of initial values of variables in the membrane labeled i.
- V_O: V_O is a set of output variables.

In this paper, we assume that V_O is included in the outermost region of the system.

We next formally define a k-th evolution program $pr_{k,i}$ as follows.

$$pr_{k,i} = \{F_{k,i}(y_{1,i}, y_{2,i}, \cdots, y_{k,i}) | e_{j,i} \to c_{k,1}|v_1 + c_{k,2}|v_2 + \cdots + c_{k,n_i}|v_{n_i}\}$$

In the above expression, $F_{k,i}(y_{1,i}, y_{2,i}, \cdots, y_{k,i})$ is called a *production function* and computed arguments $y_{1,i}, y_{2,i}, \cdots, y_{k,i} \subset V_i$, and $c_{k,1}|v_1 + c_{k,2}|v_2 + \cdots + c_{k,n_i}|v_{n_i}$ is called a *repartition protocol*. $\{v_1, v_2, \cdots, v_{n_i}\}$ is a set of variables in the region and in neighboring regions, which are outside and inside regions. In addition,

$e_{j,i}$ is called *enzyme*, where *j* is an integer running from 1 to $|V_i|$. In case that $e_{j,i} > min\{y_{1,i}, y_{2,i}, \cdots, y_{k,i}\}$, the enzyme works as a catalyst, and then, the repartition protocol allocates an output of the production function to the variables according to coefficients $\{c_{k,1}, c_{k,2}, \cdots, c_{k,n_i}\} \subset N$.

In the present paper, we assume that the EN P system is asynchronous, i.e., any numbers of applicable programs are applied in each step of computation. In other words, the asynchronous EN P system can be executed sequentially, and also can be executed maximally in parallel.

3. Compare-and-Exchange

The input of compare-and-exchange is a pair of two values *p* and *q*, and the output is also a pair of two values *x* and *y* such that $x = \min\{p,q\}$, $y = \max\{p,q\}$. An idea of Π_{COMP}, which is an asynchronous EN P system for the compare-and-exchange, is as follows. We utilize a feature that an evolution program is applied if and only if a value of an enzyme in the program is greater than all numerical variables in a production function in the program. We compare two input values *p* and *q* by setting the two values to numerical variables and enzymes. The exchange operation is executed if a value of the enzyme is smaller than a value of the numerical variable.

The following is an outline of computation of the asynchronous EN P system Π_{COMP}.

Step 1: Subtract 0.1 from a value of p to avoid tie situations.

Step 2: Set *p* to numerical variable n_1 and enzyme e_1, and set *q* to numerical variables n_2 and enzyme e_2.

Step 3: Compute a larger value *x* using the following evolution programs. (The programs are simplified to shorten the description.) Compute a smaller value *y* similarly.

$$n_1|_{e_2} \rightarrow 1|x,\ \ n_2|_{e_1} \rightarrow 1|x$$

Step 4: Output *x* and y as a result of the compare-and-exchange.

We obtained the following theorem for the proposed EN P system Π_{COMP}.

Theorem 1: The asynchronous EN P system Π_{COMP}, which executes the compare-and-exchange for two numbers, works in $O(1)$ parallel steps and $O(1)$ sequential steps, using $O(1)$ variables, $O(1)$ membranes, and $O(1)$ evolution programs.

4. Sorting

An idea of Π_{SORT}, which is an asynchronous EN P system for sorting, is based on odd-even transposition sort [4]. We assume that input of sorting is *n* values $V_0, V_1, \cdots, V_{n-1}$. An outline of the EN P system is as follows.

Step 1: Repeat the following two sub-steps, (1-1) and (1-2), $\frac{n}{2}$ times and, and them, output the values.

(1-1) Perform the compare-and-exchange for (V_{2i}, V_{2i+1}) $(0 \le i \le \frac{n}{2} - 1)$ in $\frac{n}{2}$ membranes.

(1-2) Perform the compare-and-exchange for (V_{2i-1}, V_{2i}) $(1 \le i \le \frac{n}{2} - 1)$ in $\frac{n}{2}$ membranes.

We obtained the following theorem for the proposed EN P system Π_{SORT}.

Theorem 2: The asynchronous EN P systems Π_{SORT}, which executes sorting for *n* numbers, works in $O(n)$ parallel steps and $O(n^2)$ sequential steps, using $O(n)$ variables, $O(n)$ membranes, and $O(n)$ evolution programs.

5. Conclusions

We proposed asynchronous EN P systems for compare-and-exchange and sorting. As a future work, we are considering asynchronous EN P systems using the fewer number of membranes and programs.

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